

國立虎尾科技大學九十六學年度第二學期期末考試題

班級：四電一甲 科目：工程數學(一) 時間：2008/6/27(五) 13:30-14:50

1. Find the Laplace transform of $f(t) = \begin{cases} t, & 0 \leq t < 1 \\ 2-t, & 1 \leq t < 2. \\ 0, & 2 \leq t \end{cases}$

2. Find the Laplace inverse transform of $F(s) = \ln(1 + \frac{9}{s^2})$.

3. Use the Laplace transform to solve the initial value problem

$$\frac{d^2 y}{dt^2} + 3 \frac{dy}{dt} + 2y = 20 \sin(2t), \quad y'(0) = -5, y(0) = -1.$$

4. Solve $f(t) = e^{-3t} (e^t - 5 \int_0^t f(\tau) e^{3\tau} d\tau)$ for $f(t)$.

5. Use the Laplace transform to solve the initial value problem for the following system

$$\begin{cases} x' - 2y' = -1 \\ x' + y - x = 1 \end{cases} \quad x(0) = y(0) = 0.$$

6. Expand the following function $f(x)$ in a Fourier series

$$f(x) = \begin{cases} x + \pi, & -\pi < x < 0 \\ \pi - x, & 0 \leq x < \pi \end{cases} \quad f(x) = f(x + 2\pi).$$

7. What is the Fourier series of $g(x) = \begin{cases} 1 & \text{for } -\pi < x < 0 \\ -1 & \text{for } 0 \leq x < \pi \end{cases}$ and

$$g(x) = g(x + 2\pi).$$

8. Determine the sum of the Fourier series of the function

$$f(x) = \begin{cases} \sin(2x) & \text{for } -\pi \leq x \leq 0 \\ 2 & \text{for } 0 < x \leq \pi \end{cases} \quad \text{and } f(x) = f(x + 2\pi).$$

9. Let the Fourier integral of $f(x)$ be $\int_0^\infty [A_\omega \cos(\omega x) + B_\omega \sin(\omega x)] d\omega$ and

$$f(x) = \begin{cases} -1, & \text{for } -\pi \leq x < 0 \\ 1, & \text{for } 0 \leq x \leq \pi \\ 0, & \text{for } |x| > \pi \end{cases}. \text{ Please find } A_\omega.$$

10. Find the Fourier transform of $f(x) = \begin{cases} 5 & \text{for } |x| < 2 \\ 0 & \text{for } |x| \geq 2 \end{cases}$.