國立虎尾科技大學九十七學年度第一學期期中考試題

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1. For which value(s) of k does this system $\begin{cases} x_1 + x_2 - x_3 = -2 \\ 3x_1 - 5x_2 + 13x_3 = 18 \\ x_1 - 3x_2 + 5x_3 = k \end{cases}$ have one or infinitely many solutions?
2. Find all vectors in R^3 that are orthogonal to the two vectors $\begin{bmatrix} 1 & 1 & 1 \end{bmatrix}$, and $\begin{bmatrix} 1 & 2 & 3 \end{bmatrix}$.
3. Find all solutions x_1, x_2, x_3 of the equation $\vec{b} = x_1\vec{v}_1 + x_2\vec{v}_2 + x_3\vec{v}_3$, where $\vec{b} = \begin{bmatrix} -8 \\ -1 \\ 2 \\ 15 \end{bmatrix}$, $\vec{v}_1 = \begin{bmatrix} 1 \\ 4 \\ 7 \\ 5 \end{bmatrix}$, $\vec{v}_2 = \begin{bmatrix} 2 \\ 5 \\ 8 \\ 3 \end{bmatrix}$ and $\vec{v}_3 = \begin{bmatrix} 4 \\ 6 \\ 9 \\ 1 \end{bmatrix}$.
4. Consider the system $\begin{cases} x + y = C \\ 3y + z = C \\ x + 4z = C \end{cases}$ where C is a constant. Find the smallest positive integer C such that x, y and z are all integers.
5. Find the rank of the matrix $\begin{bmatrix} a & b & c \\ 0 & d & e \\ 0 & 0 & f \end{bmatrix}$ where a, d and f are nonzero, and b, c and e are arbitrary numbers.
6. Is the vector $\begin{bmatrix} 7 & 8 & 9 \end{bmatrix}$ a linear combination of $\begin{bmatrix} 1 & 2 & 3 \end{bmatrix}$ and $\begin{bmatrix} 4 & 5 & 6 \end{bmatrix}$?
7. For which values of the constants b and c is the vector $\begin{bmatrix} 3 & b & c \end{bmatrix}$ a linear combination of $\begin{bmatrix} 1 & 3 & 2 \end{bmatrix}$, $\begin{bmatrix} 2 & 6 & 4 \end{bmatrix}$, and $\begin{bmatrix} -1 & -3 & -2 \end{bmatrix}$.
8. Please find the linear transformation matrix T if the linear transformation $T\begin{bmatrix} 5 \\ 42 \end{bmatrix} = \begin{bmatrix} 89 \\ 52 \end{bmatrix}$ and $T\begin{bmatrix} 6 \\ 41 \end{bmatrix} = \begin{bmatrix} 88 \\ 53 \end{bmatrix}$ are given.
9. For the matrix $B = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \end{bmatrix}$, find a matrix A such that $BA = I_2$. How many solutions A does this problem have?
10. Find A if $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} A \begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$.